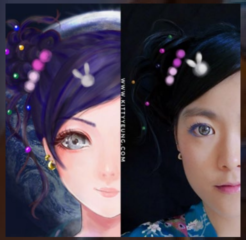
Introduction to Quantum Computing



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April 12, 2020
Hackaday, Session 3

Class structure

- Comics on Hackaday – Introduction to Quantum Computing every Wed & Sun

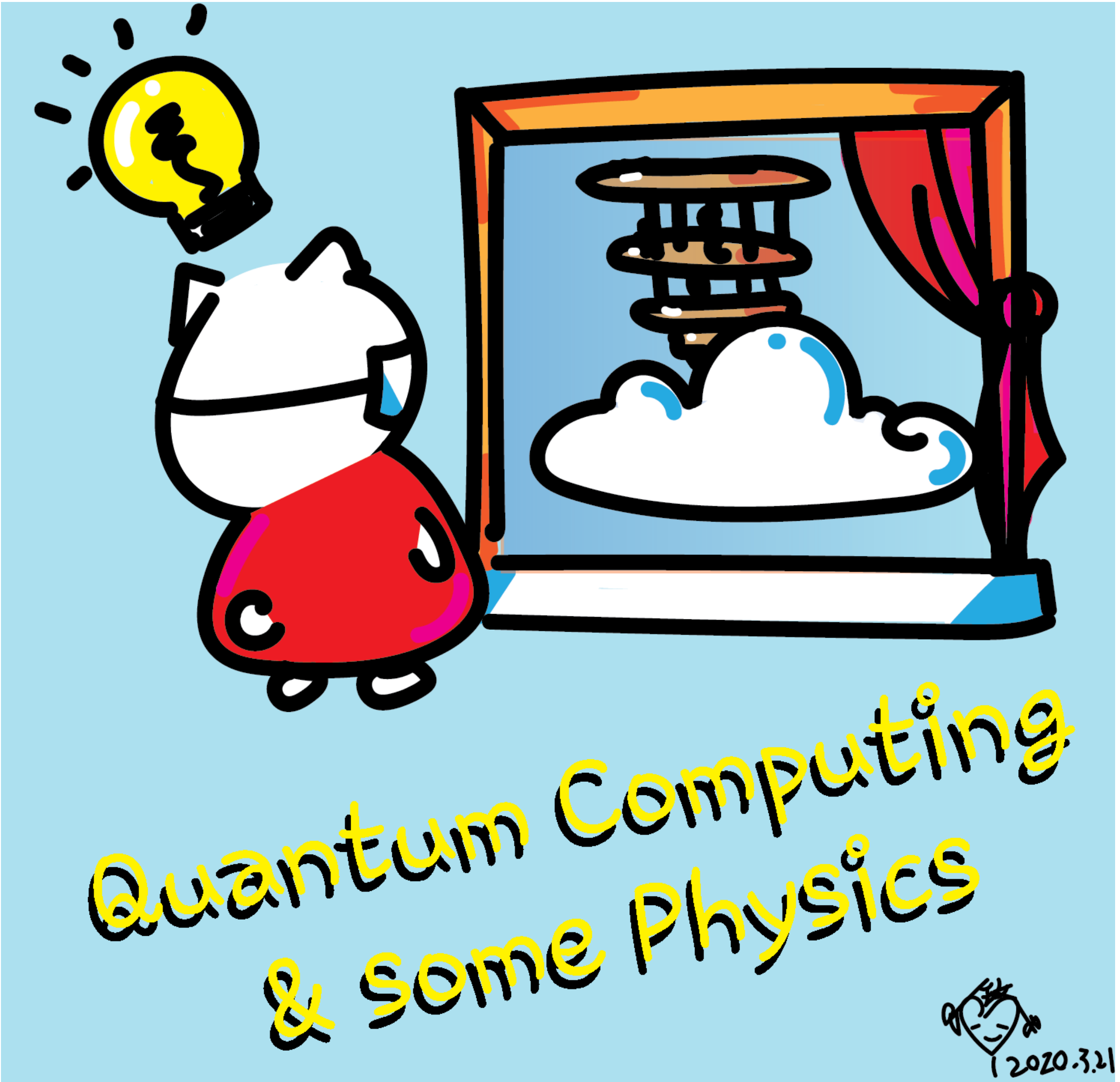
- 30 mins every Sun, one concept (theory, hardware, programming), Q&A

- Contribute to Q# documentation http://docs.microsoft.com/quantum

- Coding through Quantum Katas https://github.com/Microsoft/QuantumKatas/

- Discuss in Hackaday project comments throughout the week

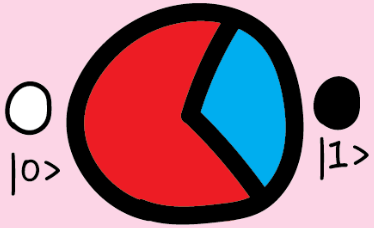
- Take notes



2020.3.28.

A qubit system is all the possible configurations in superposition.

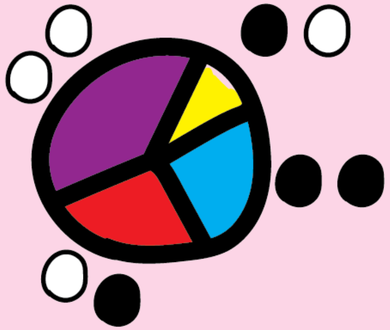
PIE CHART DENOTING PROBABILITY OF EACH CONFIGURATION



ONE QUBIT, TWO CONFIGURATIONS:

$a|0\rangle+b|1\rangle$

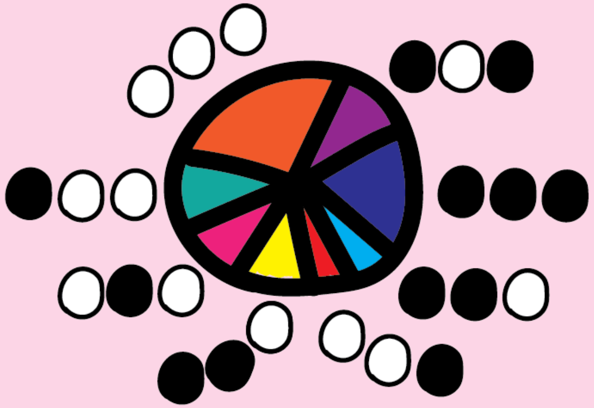
$a^2+b^2=1$ (total probability adds up to 1)



TWO QUBITS, FOUR CONFIGURATIONS:

$a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle$

$a^2+b^2+c^2+d^2=1$



THREE QUBITS, EIGHT CONFIGURATIONS:

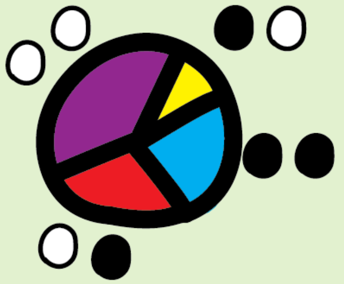
$a|000\rangle+b|001\rangle+c|010\rangle+d|100\rangle+e|110\rangle+f|101\rangle+g|011\rangle+h|111\rangle$

$a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2=1$

...

N qubits will have 2^N possible configurations in superposition!

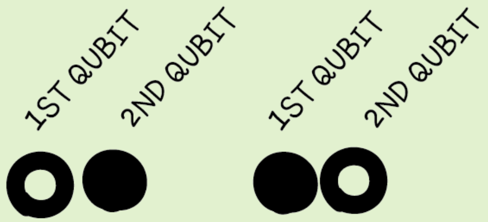
We've seen in page 9 that with two qubits, there are four possible configurations: both qubits in |0>s or |1>s, or one in |0> with the other in |1>. What if we make the |0>|0> case in superposition with the |1>|1> case? Or |0>|1> in superposition with |1>|0>?



a|0 0> + d|1 1>

If we set the system to be in this case, we know that if we measure the first qubit and get |0>, the second qubit must be in |0>, without needing to measure it.

We can also measure the second qubit to know what the first qubit is without measuring it.



b|0 1> + c|1 0>

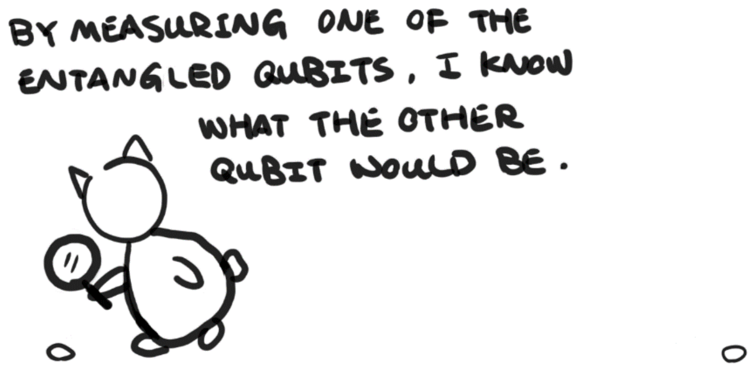
Similarly in this case, if the first qubit is |0>, the second qubit must be |1>. If the first is |1>, the second must be |0>.

The qubits are correlated. This is called "entanglement".

Entanglement

Bell states

$|\varphi^{\pm}\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$ and $|\phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$



Take $|\phi^{+}\rangle$ as an example, upon measuring the first qubit, one obtains two possible results:

1. First qubit is 0, get a state $|\phi'\rangle = |00\rangle$ with probability ½.
2. First qubit is 1, get a state $|\phi''\rangle = |11\rangle$ with probability ½.

If the second qubit is measured, the result is the same as the above. This means that measuring one qubit tells us what the other qubit is.

Entanglement

Math insert – entangled states cannot be factored back to individual qubits--------------

Remember in section 1.1, a two-qubit state can be obtained by doing a tensor product of two individual one-qubit states. However, a Bell state cannot be factored back into two individual qubits. For example,

$$|\phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}.$$

If we want to factor it back to two separate qubits as in $\binom{a}{b} \otimes \binom{c}{d}$, then this set of equations need to be simultaneously satisfied

$ac = \frac{1}{\sqrt{2}}$, $ad = 0, bc = 0$ and $bd = \frac{1}{\sqrt{2}}$. Unfortunately, it is impossible. This set of equations has no solution. It can only be 50% chance of getting $|00\rangle = \binom{1}{0} \otimes \binom{1}{0}$ or $|11\rangle = \binom{0}{1} \otimes \binom{0}{1}$.

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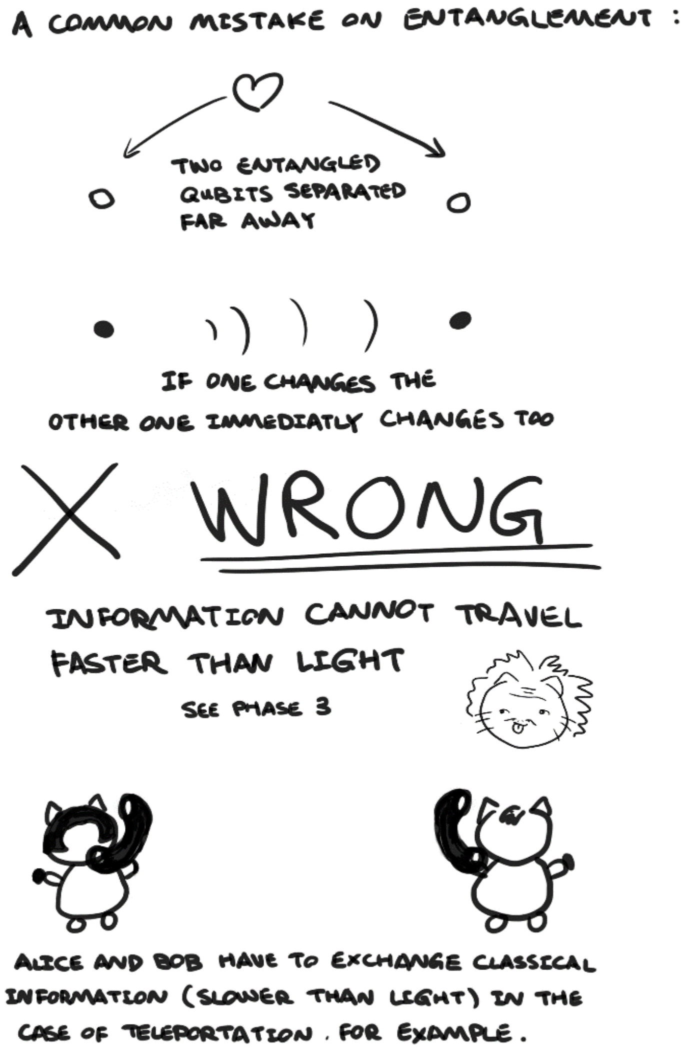
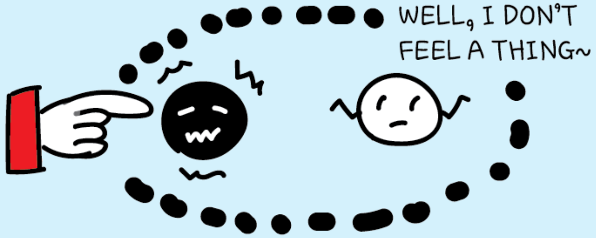
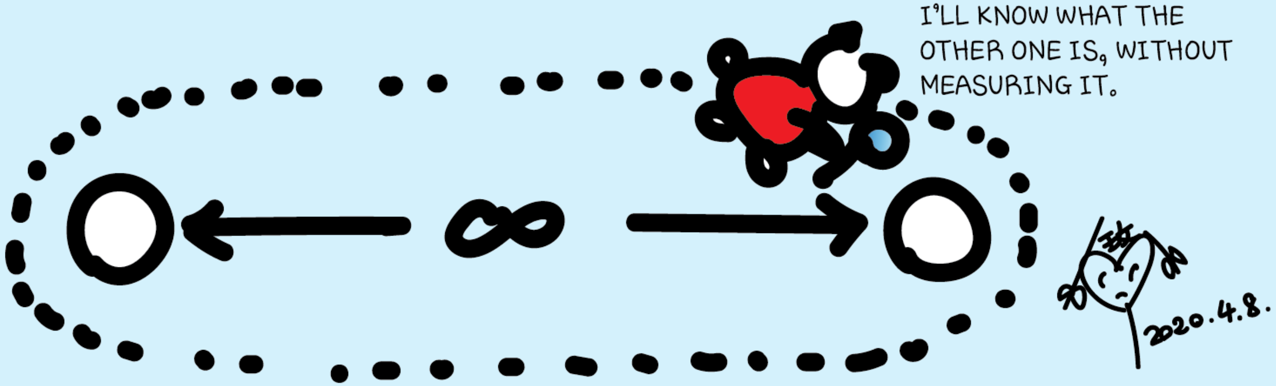
https://quantumfactsheet.github.io/

A common mistake





When we change one of the entangled qubits, the other qubit does not instantaneously change. (That would imply faster-than-light information transfer, which is prohibited. This is a common mistake people make when talking about entanglement.)

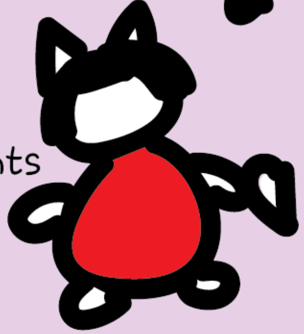


They can remain entangled even if they are separated infinitely far apart. There is no "spooky" interaction between them. All it means is that their measurement results are correlated. And entanglement simply does not depend on distance.

We can use entanglement to our advantage, such as in communication or encryption.

First prepare a Bell state, e.g. $(|01\rangle+|10\rangle)/\sqrt{2}$

ALICE

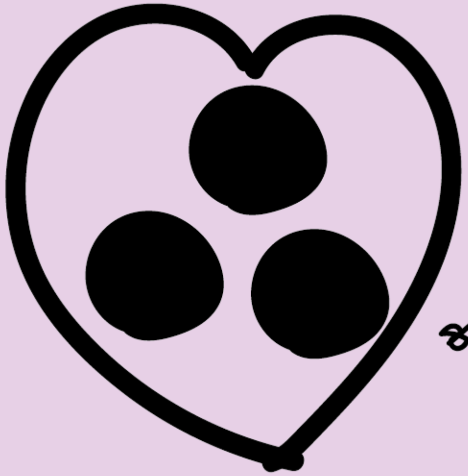


If Alice measures and gets $|0\rangle$, she knows Bob will get $|1\rangle$. If she wants him to get $|0\rangle$, She'll ask him to flip his qubit.

Give the 1st qubit to Alice, and the 2nd to Bob.

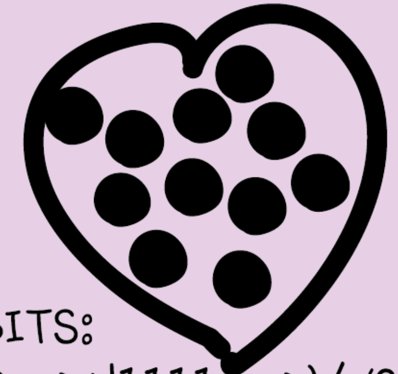
BOB

Of course, entanglement can happen between any number of qubits. The multi-qubit counterpart of Bell states are called the Greenberger–Horne–Zeilinger (GHZ) states.

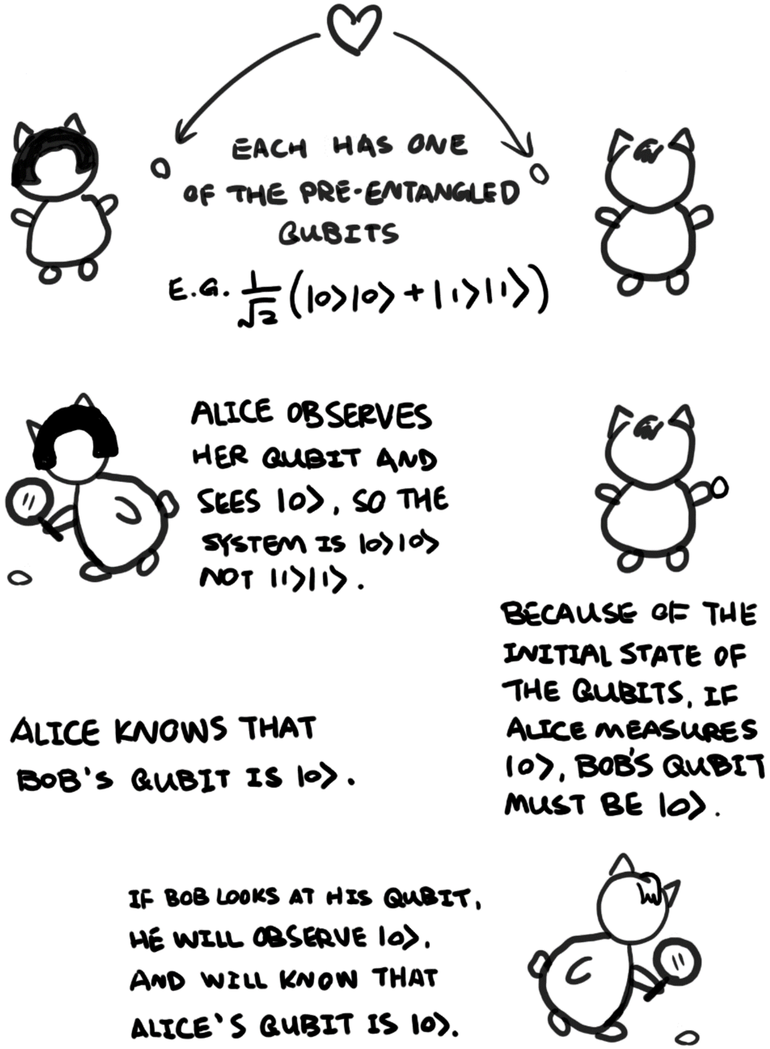


THREE QUBITS:
$(|000\rangle+|111\rangle)/\sqrt{2}$

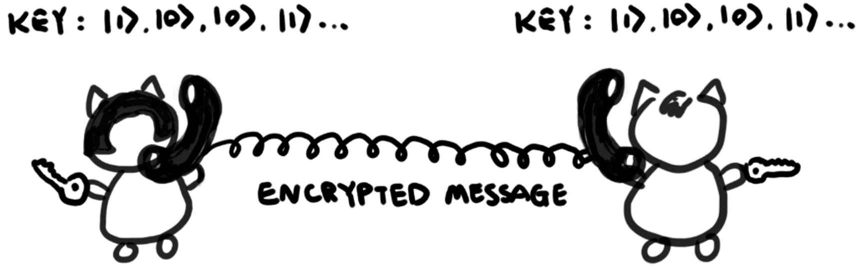
2020.4.11.



N QUBITS:
$(|0000\ldots\rangle+|1111\ldots\rangle)/\sqrt{2}$
$=(|0\rangle^{\otimes N}+|1\rangle^{\otimes N})/\sqrt{2}$

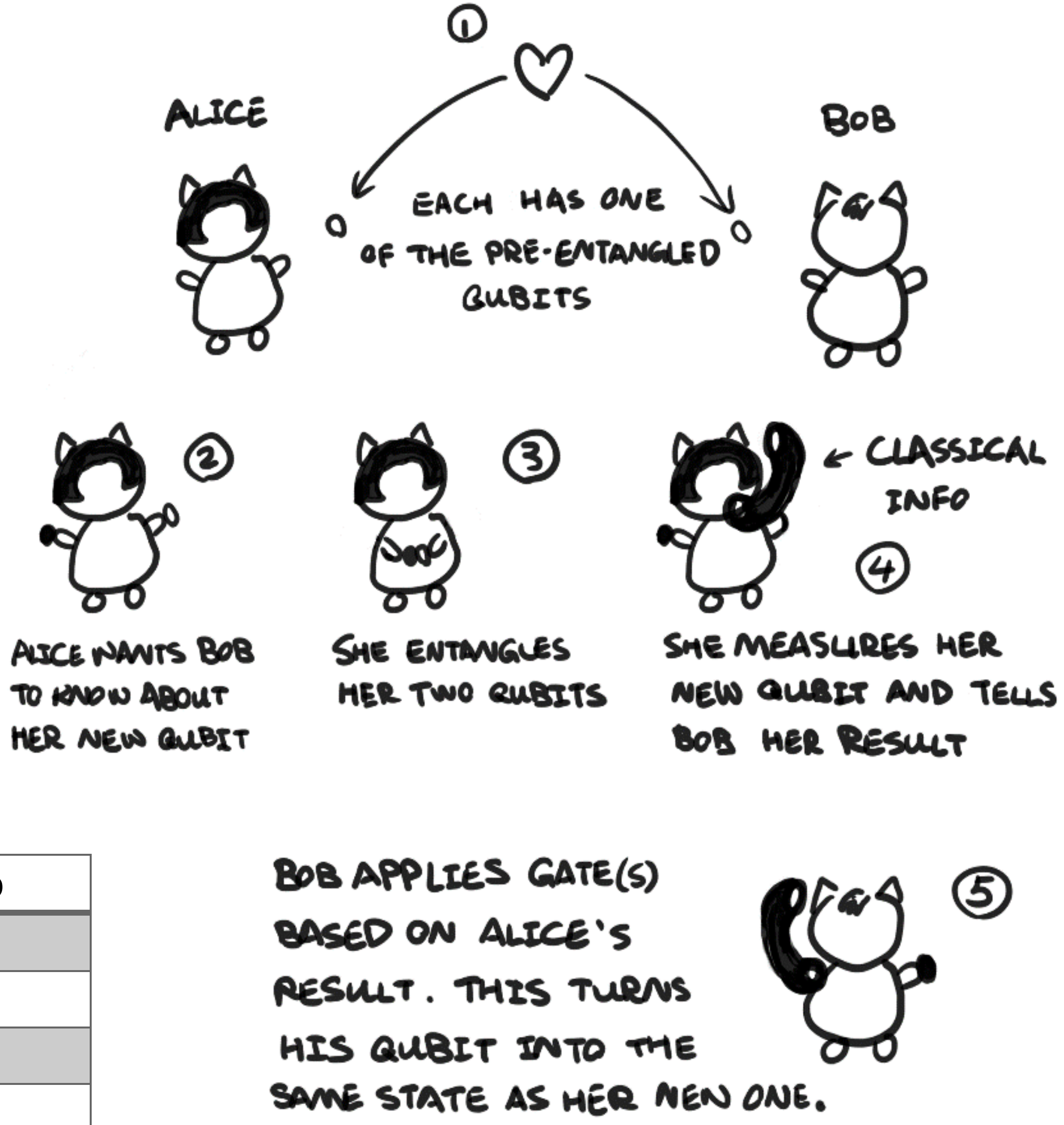


Encryption



They can't communicate faster than light, but at least they can communicate securely.

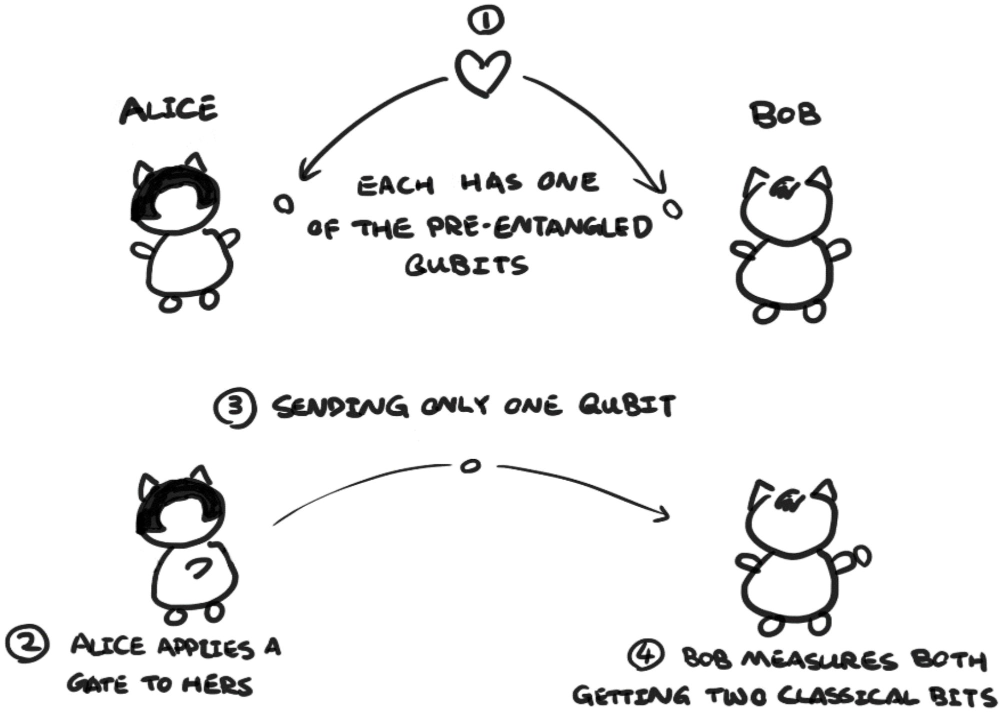
Teleportation



First two qubits	Third qubit	Alice tells Bob to
00	$[\alpha\|0\rangle + \beta\|1\rangle]$	do nothing
01	$[\alpha\|1\rangle + \beta\|0\rangle]$	apply X
10	$[\alpha\|0\rangle - \beta\|1\rangle]$	apply Z
11	$[\alpha\|1\rangle - \beta\|0\rangle]$	apply X and Z

Superdense coding

$$|\phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$



To send ‘01’, she applies an X gate $$|\varphi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

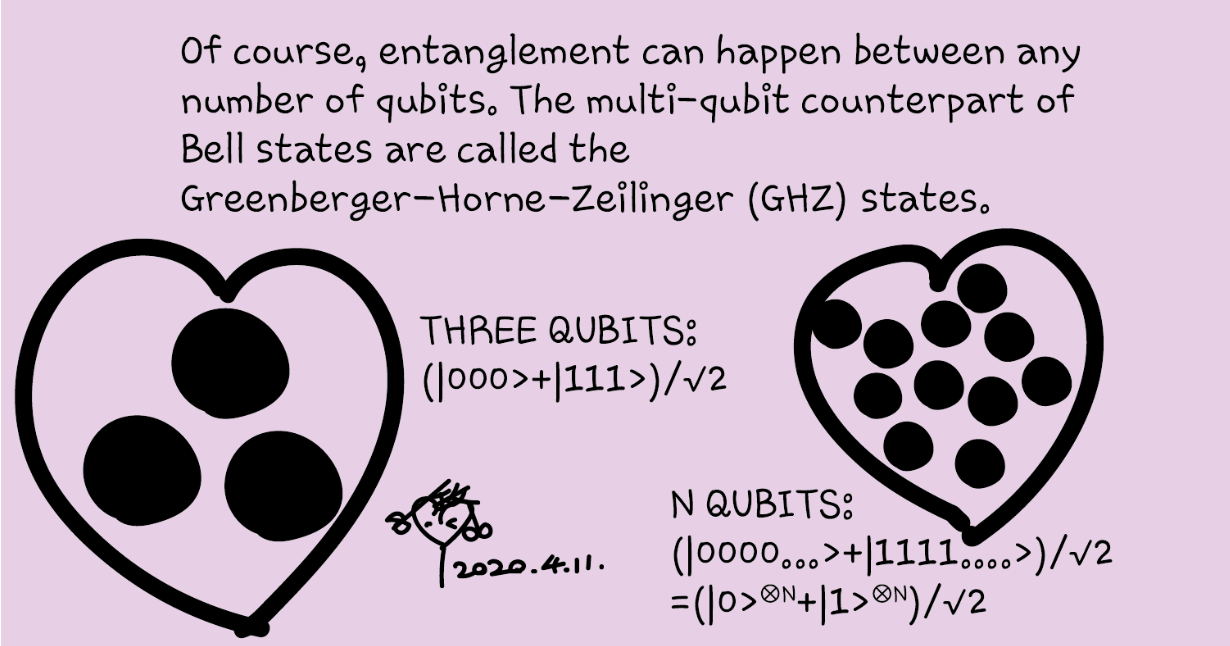
To send ‘10’, she applies a Z gate $$|\phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

For ‘11’, she uses an iY gate or a Z * X gate $$|\varphi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Greenberger – Horne – Zeilinger (GHZ) states

$$|GHZ\rangle_{simplest} = \frac{|000\rangle + |111\rangle}{\sqrt{2}}$$

$$|GHZ\rangle_{general} = \frac{|0\rangle^{\otimes \mathrm{N}} + |1\rangle^{\otimes \mathrm{N}}}{\sqrt{2}}$$



Imagine N = 500, there are 2^{500} possible states in the system - more than the number of atoms in the Universe.

However, entanglement can be disadvantageous, too.

If the qubits are not perfectly isolated,

entanglement with their environment can easily happen, causing the qubits to **decohere** from each other.

Measurements also cause decoherence, when the measuring device acts as the environment that entangles with the qubits.

Therefore, measurements must be delicately done. Otherwise, they cause errors.